TEMPERATURE DISTRIBUTION IN THE HUMAN BODY UNDER GENERAL DEEP HYPERTHERMIA

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A mathematical model is proposed for the distribution of temperature in the human body under general deep hyperthermia.

One of the most promising methods for the treatment of certain forms of malignant tumors is hyperoxidation–hyperthermia which was first introduced into clinical practice in the USSR at the Scientific Research Institute for Oncology and Medial Radiology of the Ministry of Public Health, Belorussian SSR. One of the basic parts of this method is hyperthermia, i.e., heating the body to high temperatures (40-41°) and maintaining these temperatures for several hours [1, 2].

To carry out this procedure safely, it is necessary that the brain temperature be 1-1.5° lower than that of the "core" of the body. This is achieved by supplementary cooling of the head.

We have attempted to study the dynamics of temperature variation in the body for surface heating of the trunk and cooling of the head.

The temperature distribution in a uniform physical body is given by the solution of the equation of thermal conductivity under various boundary conditions. In the general case, the equation of thermal conductivity has the following form [4],

$$c\gamma \frac{\partial t}{\partial \tau} = \operatorname{div}[\lambda \operatorname{grad} t] + w$$

(w is the specific intensity of internal heat sources, W/m^3; γ is the density of the body, kg/m^3; $a = \lambda / c\gamma$ is the coefficient of thermal diffusivity, m^2/sec).

However, the use of this equation for a solution of our problem is impossible because of the pronounced nonuniformity of the various tissues, i.e., $c = c(x, y, z)$, $\gamma = \gamma(x, y, z)$, $a = a(x, y, z)$, $w = w(x, y, z)$, the existence of effective thermoregulation, i.e., $a = a(t)$, $w = w(t)$, and the complex geometric shape.

For solution of the problem formulated, we therefore arbitrarily divide the body into layers (zones) and set up the equation of thermal balance for each layer (zone) assuming that within the boundaries of a layer (zone) c, γ, a, and w are independent of the coordinates, and c, γ, and a are also independent of temperature. The choice of the number of zones and the subdivision of the body into zones are determined by the specific conditions of the problem to be solved and by the permissible error of the solution. For our purposes, it is sufficient to define four zones: the skin, "core" of the body, the brain, and the brain covering. We also make a few other assumptions. We assume the main mechanism for heat transfer in the body is a convectional mechanism associated with movement of the blood; the volume rate of blood flow depends linearly on temperature; the blood instantaneously takes on the temperature of the zone in which it is; w is a known function of temperature within the boundaries of a zone.

The model we have proposed is sketched in Fig. 1 in general form.

As is clear from the figure, the process of heat transfer takes place in the following manner. The heat flux incident on the skin heats the blood circulating in it to the temperature t_s. The blood flowing

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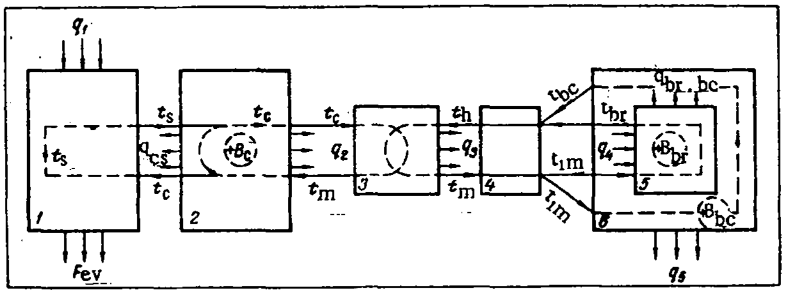

Fig. 1. Schematic diagram of superheating of the body: 1) skin; 2) "core" of the body; 3) heart; 4) neck; 5) brain; 6) brain covering.

through the "core" of the body takes on the temperature t_c, releasing or acquiring a certain amount of heat in this instance. Then blood at the temperature t_c enters a heat exchanger, the role of which is played by the heart–lung system, where it mixes with blood flowing from the head at a temperature t_h which is the result of mixing of blood from the brain at a temperature t_{br} and blood from the brain covering at a temperature t_{bc}. Blood at the stable temperature t_m emerging from the heat exchanger divides into two flows. One portion enters the head where it once again divides into two flows, one of which enters the brain and the other enters the brain covering (note that this portion of the blood can be subjected to additional cooling while passing through the neck). The second portion of the blood returns to the body where it once again divides into two flows; one of them enters the "core" of the body and the other, passing through the "core," enters the skin.

We assume that the prime purpose of the thermoregulatory system is maintenance of a constant temperature in the "core" of the body. This is achieved both by redistribution of the intensity of blood flow between zones (blood flow through the skin is significantly intensified) [3] and by the initiation of certain heat-transfer mechanisms (increased sweating in people, panting in dogs and certain other animals). It was shown [5, 6] that these heat-transfer mechanisms are Rayleigh functions of time, i.e., they are small if $t_c < t_c^{st}$ and large if $t_c \geq t_c^{st}$.

Taking all this into account, the system of equations describing this model appears to be:

$$m_s c_s t'_s = q_1 + q_{cs} - F_{ev},$$
$$m_c c_c t'_c = B_c - q_{cs} - q_2,$$
$$m_{br} c_{br} t'_m = B_{br} + q_3 - q_{br.bc}$$
$$m_{bc} c_{bc} t'_{bc} = B_{bc} + q_4 - q_5 + q_{br.bc}$$
$$t'_1 = f_1(\tau), \quad t'_2 = f_2(\tau),$$

where

$$q_1 = \alpha_1 S_s (t_1 - t_s);$$
$$q_2 = (V_{bl} - V_{bl.h})(t_m - t_c)\gamma_{bl} c_{bl};$$
$$q_3 = V_{bl.br}\gamma_{bl} c_{bl}(t_{lm} - t_{br});$$
$$q_4 = V_{bl.bc}\gamma_{bl} c_{bl}(t_{lm} - t_{bc});$$
$$q_5 = \alpha_2 S_{bc}(t_{bc} - t_2);$$
$$q_{cs} = V_{bl.s}\gamma_{bl} c_{bl}(t_c - t_s);$$
$$q_{br.bc} = \frac{\lambda S_{br}}{l}(t_m - t_{br});$$
$$B_c = B_{c0}[1 + \beta_1(t_c - t^*)];$$
$$B_{br} = B_{br.0}[1 + \beta_3(t_{br} - t^*)];$$
$$M_{bl} = V_{bl}\gamma_{bl};$$
$$V_{bl} = V_{bl.0}[1 + \beta_2(t_c - t^*)];$$
$$V_{bl.br} = V_{bl.br.0}[1 + \beta_4(t_{br} - t^*)];$$
$$V_{bl.s} = V_{bl.s.0}[1 + \beta_5(t_s - 33)];$$
$$M_{bl.h} = M_{bl.br} + M_{bl.bc};$$

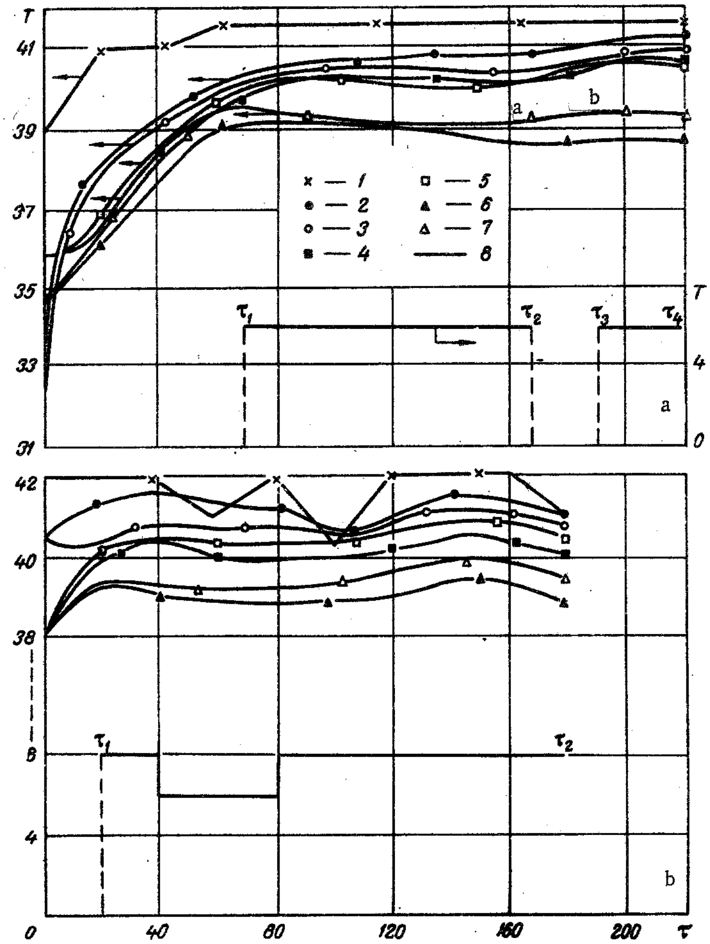

Fig. 2. Dynamics of body temperature during general deep hyperthermia [a) patient I; b) patient S]: 1) hot-water temperatures; 2) skin temperature; 4) temperature of body "core"; 6) brain temperature; temperature variation at these same points [3) skin; 5) "core"; 7) brain] calculated from the proposed model; 8) cold-water temperature. t, °C; τ, min.

$$M_{bl} = \sum_i M_{bl.i}, \quad i = s,\ c,\ br,\ bc;$$

$$t_m = \frac{1}{M_{bl}} [t_c (M_{bl} - M_{bl.h}) + t_{br} M_{bl.br} + t_{bc} M_{bl.bc}]; \qquad \text{(II)}$$

$$t_m = t_m - \Delta t, \text{ where } \Delta t = \frac{t_m - t_3}{M_{bl.h} c_{bl}} S_n \alpha_3.$$

The subscripts s, c, br, bc, bl, and n indicate assignment of the corresponding parameter to the skin, the "core," the brain, the brain covering, the blood, and the neck respectively.

Values of the coefficients β_1, β_2, β_3, β_4, β_5, and F_{ev} are defined in the following manner:

$$\beta_i = \begin{cases} \beta_i' & i = 1,\ 2,\ t_c \geqslant t^*, \\ \beta_i'' & ,\ t_c < t^*, \end{cases}$$

$$\beta_j = \begin{cases} \beta_j', & t_{br} \geqslant t^*, \\ \beta_j'', & t_{br} < t^*,\ j = 3,\ 4, \end{cases}$$

$$\beta_5 = \begin{cases} \beta_5', & t_s \geqslant 33, \\ \beta_5'', & t_s < 33, \end{cases}$$

$$F_{ev}=\begin{cases}F'_{ev}, & t_c>t^*,\\ F''_{ev}, & t_c\leqslant t^*.\end{cases} \tag{III}$$

Keeping the relations (II) and (III) in mind, we reduce the equation system (I) to a somewhat different form:

$$\begin{aligned}
t'_s &= \frac{1}{m_s c_s}\left[\alpha_1 S_s(t_1-t_s)+M_{bl.s}c_{bl}(t_c-t_s)-F_{ev}\right],\\
t'_c &= \frac{1}{m_c c_c}\{B_{c0}[1+\beta_1(t_c-t^*)]+(M_{bl}-M_{bl.h})(t_m-t_c)c_{bl}-M_{bl.s}c_{bl}(t_c-t_s)\},\\
t'_{br} &= \frac{1}{m_{br}c_{br}}\left\{B_{br0}[1+\beta_2(t_{br}-t^*)]+M_{bl.br}c_{bl}(t_m-t_{br})-\frac{\lambda S_{br}}{l}(t_{br}-t_{bc})\right\},\\
t'_{bc} &= \frac{1}{m_{bc}c_{bc}}\left[B_{bc}+M_{bl.bc}c_{bc}(t_m-t_{bc})+\frac{\lambda S_{br}}{l}(t_{br}-t_{bc})+\alpha_2 S_{bc}(t_{bc}-t_2)\right],\\
t'_1 &= f_1(\tau),\quad t'_2=f_2(\tau).
\end{aligned} \tag{IV}$$

The initial conditions at $\tau = 0$ are

$$\begin{aligned}
t_s &= t_{s0}, & t_{bc} &= t_{bc.0}\\
t_c &= t_{c0}, & t_1 &= t_{10},\\
t_{br} &= t_{br0}, & t_2 &= t_{20}.
\end{aligned} \tag{V}$$

The equation system (IV) under the initial conditions (V) was solved on a computer and the results compared with experimental data obtained at the Scientific Research Institute for Oncology and Medical Radiology of the Ministry of Public Health, Belorussian SSR (Fig. 2).

Experimental and theoretical time dependences of the skin, "core," and head temperatures of an individual are shown in Fig. 2 for certain modes of hyperthermia. A comparison of theoretical and experimental data shows that the model satisfactorily describes the distribution of temperature in the body of an individual, particularly for slow variations in the temperature of the heating or cooling medium. For abrupt changes in the temperature of the heating or cooling medium, the difference between the experimental and theoretical results is intensified (segment ab of curve 7 in Fig. 2a) since the condition requiring linear dependence of volume rate of blood flow on temperature breaks down.

The sufficiently satisfactory agreement of theoretical and experimental curves makes it possible to plan the hyperthermia procedure depending upon the individual characteristics of the patient.

NOTATION

$t'_i = dt_i/d\tau$; t	is the temperature, °C;
t*	is the standard temperature, °C;
m	is the mass, kg;
c	is the specific heat, J/kg·deg;
V_{bl}	is the bulk velocity of blood, m^3/sec;
α_1, α_2, α_3	are the heat transfer coefficients between skin and hot heat-transfer agent, cold membrane of head and cold heat-transfer agent and between neck-cooling medium and blood through neck, W/m^2·deg;
K	is the thermal conductivity, W/m·deg;
B_0	is the basal metabolism, W;
F_{ev}	is the rate of evaporation, W;
S	is the surface, m^2;
l	is the thickness of head membrane, m;
t_1, t_2, t_3	are the temperatures of hot water, cold water and neck cooling medium, °C.

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